



**GCE**

**Further Mathematics B (MEI)**

**Y435/01: Extra pure**

Advanced GCE

**Mark Scheme for Autumn 2021**

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## Annotations and abbreviations

Annotation in scoris	Meaning
✓ and *	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction <b>In this question you must show detailed reasoning</b> appears in the question.

Question		Answer	Marks	AO	Guidance	
1	(a)	<p>DR</p> $z = f(2, y) = 8 + 4y - 2y^2$ $= 10 - 2(y - 1)^2 \Rightarrow \text{max at } (1, 10) \text{ or } (2, 1, 10)$  <p>Crossing <math>z</math>-axis at 8, <math>y</math>-axis at <math>1 \pm \sqrt{5}</math> and showing (1, 10) as a max</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>A1</b></p> <p><b>[4]</b></p>	<p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p>	<p>Deriving correct equation of graph of section.</p> <p>Finding TP by completing the square, use of “<math>-b/2a</math>”, differentiation or mid-point between roots.</p> <p><math>\cap</math>-shaped parabola which <u>crosses</u> horizontal axis twice.</p> <p>Coordinates of intercepts and max must be shown on graph or apparent in working. Allow decimal values (awrt <math>-1.2</math> and <math>3.2</math>) for the <math>y</math>-intercepts.</p>	<p>Working must be shown.</p> <p>Condone incorrect variable names on axes (eg <math>x</math>-<math>y</math> for <math>y</math>-<math>z</math>).</p> <p><math>z</math> intercept must be shown as positive and max in 1<sup>st</sup> quadrant. However, scale is unimportant except that the negative <math>y</math>-intercept must be closer to <math>O</math> than the positive one.</p>

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	(b)	$\frac{\partial z}{\partial x} = 3x^2 + 2xy$ $\frac{\partial z}{\partial y} = x^2 - 4y$ $\frac{\partial z}{\partial x} = 0$ $\Rightarrow 3x^2 + 2xy = 0$ $\Rightarrow \text{either } x = 0 \text{ or } x = -\frac{2}{3}y \text{ or } y = -\frac{3}{2}x$ $\frac{\partial z}{\partial y} = 0, x = -\frac{2}{3}y \Rightarrow 4y = \frac{4}{9}y^2 \Rightarrow y = 9$ $x = -6$ $z = -54 \text{ so } (-6, 9, -54)$ $\dots \text{ or } x = 0 \Rightarrow y = 0 \text{ so } (0, 0, 0)$	<b>B1</b>	<b>1.1</b>		
			<b>B1</b>	<b>1.1</b>		
			<b>M1</b>	<b>1.1</b>	Setting a partial derivative to 0 and deriving condition(s) on $x$ and/or $y$ .	Or $\frac{\partial z}{\partial y} = 0$ $\Rightarrow y = \frac{1}{4}x^2 \text{ or } x^2 = 4y \text{ or } x = \pm 2\sqrt{y}$
			<b>M1</b>	<b>1.1</b>	Substituting condition into other partial derivative equation to derive a non-zero value for $x$ or $y$ . $\frac{\partial z}{\partial y} = 0, y = -\frac{3}{2}x$ $\Rightarrow x^2 + 6x = 0 \Rightarrow x = -6\dots$	$\frac{\partial z}{\partial x} = 0, y = \frac{1}{4}x^2$ $3x^2 + \frac{1}{2}x^3 = 0 \Rightarrow x = -6$ or $\frac{\partial z}{\partial x} = 0, x = \pm 2\sqrt{y}$ $12y \pm 4y^{\frac{3}{2}} = 0 \Rightarrow y = 9$
			<b>A1</b>	<b>1.1</b>	From correct working only.	$y = 9$
			<b>A1</b>	<b>1.1</b>	Derived from both $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ (could be by observation).	If an extra SP is presented then <b>A1</b> can be awarded for either SP correct and then <b>A0</b> .
			<b>[7]</b>			

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2	(a)	From Lagrange's Theorem the order of any subgroup of $G$ must be a factor of 8 and 6 is not a factor of 8	<b>B1</b>  <b>[1]</b>	<b>2.4</b>	Or "order of any subgroup of $G$ (or an group of order 8) must be 1, 2 or 4 (or 8)" or "order of any subgroup must be a factor of the order of the group and 6 is not a factor of 8".	If referenced, Lagrange's Theorem does not have to be quoted provided that it is applied. So <b>B1</b> for eg "6 is not a factor of 8 so by Lagrange's Theorem there can be no subgroup of $G$ of order 6" but <b>B0</b> for eg "By Lagrange's Theorem there can be no subgroup of $G$ of order 6".
2	(b)	$g^2$ (or $g^6$ )  $g^6$ (or $g^2$ ) and no other	<b>B1</b>  <b>B1</b> <b>[2]</b>	<b>2.2a</b>  <b>2.2a</b>		May see eg $gg$ or $g \circ g$ used here and/or throughout. Allow any multiplicative notation and any symbol for a binary operation.
2	(c)	$e \leftrightarrow 0$ $g \leftrightarrow 1, g^2 \leftrightarrow 2, g^3 \leftrightarrow 3, g^4 \leftrightarrow 4, g^5 \leftrightarrow 5, g^6 \leftrightarrow 6, g^7 \leftrightarrow 7$ $g \leftrightarrow 3, g^2 \leftrightarrow 6, g^3 \leftrightarrow 1, g^5 \leftrightarrow 7, g^6 \leftrightarrow 2, g^7 \leftrightarrow 5$ $g \leftrightarrow 5, g^2 \leftrightarrow 2, g^3 \leftrightarrow 7, g^5 \leftrightarrow 1, g^6 \leftrightarrow 6, g^7 \leftrightarrow 3$ and $g \leftrightarrow 7, g^2 \leftrightarrow 6, g^3 \leftrightarrow 5, g^5 \leftrightarrow 3, g^6 \leftrightarrow 2, g^7 \leftrightarrow 1$	<b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b>	<b>2.2a</b> <b>2.2a</b> <b>2.2a</b> <b>2.2a</b>	Only needs to be seen once. Any one. Any other. Other two. Ignore repeats.	$g^4 \leftrightarrow 4$ does need not be seen again $g^4 \leftrightarrow 4$ does need not be seen again
		<b>Alternative method:</b> $e \leftrightarrow 0$ Either $g \leftrightarrow 1$ or $g \leftrightarrow 3$ or $g \leftrightarrow 5$ or $g \leftrightarrow 7$  $g \leftrightarrow 1, g^2 \leftrightarrow 2, g^3 \leftrightarrow 3, g^4 \leftrightarrow 4, g^5 \leftrightarrow 5, g^6 \leftrightarrow 6, g^7 \leftrightarrow 7$  $g \leftrightarrow 3, g^2 \leftrightarrow 6, g^3 \leftrightarrow 1, g^5 \leftrightarrow 7, g^6 \leftrightarrow 2, g^7 \leftrightarrow 5$ and $g \leftrightarrow 5, g^2 \leftrightarrow 2, g^3 \leftrightarrow 7, g^5 \leftrightarrow 1, g^6 \leftrightarrow 6, g^7 \leftrightarrow 3$ and $g \leftrightarrow 7, g^2 \leftrightarrow 6, g^3 \leftrightarrow 5, g^5 \leftrightarrow 3, g^6 \leftrightarrow 2, g^7 \leftrightarrow 1$	<b>B1</b> <b>M1</b>  <b>A1</b>  <b>A1</b>		Only needs to be seen once Giving all 4 possible isomorphism options for any generator of $G$ (ie $g, g^3, g^5$ or $g^7$ ) Completing the specification of any one isomorphism Other three. Ignore repeats.	$g^4 \leftrightarrow 4$ does need not be seen again
			<b>[4]</b>			

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3	(a)	$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & 3 & 0 \\ 0 & 2-\lambda & 2 \\ 1 & 3 & 4-\lambda \end{vmatrix}$ $= (3-\lambda)[(2-\lambda)(4-\lambda) - 2 \times 3] - 3(0 - 2 \times 1) \text{ oe}$ $= -\lambda^3 + 9\lambda^2 - 20\lambda + 12 = 0$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b> <b>[3]</b></p>	<p><b>1.1a</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p>	<p>Formation of appropriate determinant soi.</p> <p>Attempt to expand determinant. Allow one slip.</p> <p>Must be an equation. ISW.</p>	<p>May see eg expansion by 1st col:  <math>(3-\lambda)[(2-\lambda)(4-\lambda) - 6] + 1(6-0)</math>  Or other formulation eg:  <math>((3-\lambda)(2-\lambda)(4-\lambda) + 6 + 0)</math>  <math>-(0 + 6(3-\lambda) + 0)</math></p>
3	(b)	1, 2 and 6 substituted into (a) equation to verify	<b>B1</b> <b>[1]</b>	<b>1.1</b>	eg checking trace is insufficient.	
3	(c)	$3a + 3b = a$ or $2a$ or $6a$ and $2b + 2c = b$ or $2b$ or $6b$ and $a + 3b + 4c = c$ or $2c$ or $6c$  $\lambda = 1: 2a = -3b, b = -2c$ or $\lambda = 2: c = 0, a = -3b$ or $\lambda = 6: a = b, c = 2b$  $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$  $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>[4]</b></p>	<p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p>	<p>Correctly forming 3 equations in 3 unknowns for one of their eigenvalues. May see explicit choice of eg <math>c = 1</math> to form 3 equations in 2 unknowns.</p> <p>Attempt to solve equations for at least one of their eigenvalues leading to two unknowns in terms of 3<sup>rd</sup>.</p> <p>or any non-zero multiple.</p> <p>or any non-zero multiple.</p>	<p>Or formation of appropriate determinant eg <math>\begin{vmatrix} \mathbf{i} &amp; \mathbf{j} &amp; \mathbf{k} \\ -2 &amp; 4-\lambda &amp; -4 \\ 0 &amp; -1 &amp; 3-\lambda \end{vmatrix}</math>.</p> <p>Attempt to expand determinant (might be in terms of <math>\lambda</math>) eg <math>\begin{pmatrix} 8-7\lambda+\lambda^2 \\ 6-2\lambda \\ 2 \end{pmatrix}</math>. Can be inferred by 2 correct coefficients.</p>

<b>3</b>	<b>(d)</b>	$\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$ $\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}^{-1} = \frac{1}{10} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} \text{ oe}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix}$ $\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} \frac{1}{10} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} =$ $\begin{pmatrix} -2 & -6 & 4 \\ -5 \times 2^n & -5 \times 2^n & 5 \times 2^n \\ 6^n & 3 \times 6^n & 3 \times 6^n \end{pmatrix}$ $\frac{1}{10} \begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -2 & -6 & 4 \\ -5 \times 2^n & -5 \times 2^n & 5 \times 2^n \\ 6^n & 3 \times 6^n & 3 \times 6^n \end{pmatrix} =$ $\frac{1}{10} \begin{pmatrix} -6 + 15 \times 2^n + 6^n & -18 + 15 \times 2^n + 3 \times 6^n & 12 - 15 \times 2^n + 3 \times 6^n \\ 4 - 5 \times 2^n + 6^n & 12 - 5 \times 2^n + 3 \times 6^n & -8 + 5 \times 2^n + 3 \times 6^n \\ -2 + 2 \times 6^n & -6 + 6^{n+1} & 4 + 6^{n+1} \end{pmatrix}$	<b>M1</b>	<b>3.1a</b>	Forming matrix of their eigenvectors, <b>E</b> .	May be in decimal form: $\begin{pmatrix} -0.2 & -0.6 & 0.4 \\ -0.5 & -0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 \end{pmatrix}$  $\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} =$ or $\begin{pmatrix} 3 & -3 \times 2^n & 6^n \\ -2 & 2^n & 6^n \\ 1 & 0 & 2 \times 6^n \end{pmatrix}$  $\frac{1}{10} \begin{pmatrix} 3 & -3 \times 2^n & 6^n \\ -2 & 2^n & 6^n \\ 1 & 0 & 2 \times 6^n \end{pmatrix} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} =$ etc. Condone $6 \times 6^n$ unsimplified.
			<b>A1FT</b>	<b>3.1a</b>	BC. Finding inverse of their matrix of eigenvectors.	
			<b>B1</b>	<b>3.1a</b>	Matrix of eigenvalues must be consistent with matrix of eigenvectors. Allow $1^n$ .	
			<b>M1</b>	<b>3.1a</b>	Forming $\mathbf{EA}^n\mathbf{E}^{-1}$ . Can be awarded if $\mathbf{A}^n$ incorrect or uncalculated but eigenvectors must be in same order as eigenvalues.	
			<b>M1</b>	<b>1.1</b>	Proper attempt to multiply either the first two or the last two (of 3) in the correct order (with or without $\frac{1}{10}$ ).	
			<b>A1</b>	<b>1.1</b>	or	
<b>[6]</b>						



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4	(a)	<p>CF: <math>u_{n+2} - 3u_{n+1} - 10u_n = 0</math> and <math>u_n = \alpha r^n</math>  <math>\Rightarrow r^2 - 3r - 10 = 0</math>  <math>\Rightarrow r = 5</math> or <math>r = -2</math>            CF is <math>\alpha 5^n + \beta(-2)^n</math></p> <p>Trial function: <math>u_n = an + b</math></p> <p><math>a(n+2) + b - 3[a(n+1) + b] - 10(an + b)</math>  <math>= 24n - 10</math>  <math>\Rightarrow (a - 3a - 10a) = 24</math>            and <math>2a + b - 3a - 3b - 10b = -10</math></p> <p><math>a = -2</math> and <math>b = 1</math> so GS is  <math>u_n = 1 - 2n + \alpha 5^n + \beta(-2)^n</math></p>	<p><b>M1</b></p> <p><b>A1FT</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>[6]</b></p>	<p><b>1.1a</b></p> <p><b>1.1</b></p> <p><b>1.1a</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p>	<p>Deriving the auxiliary equation (allow one sign error).</p> <p>FT correct roots of their AE to form CF (do not ISW).</p> <p>Correct form.</p> <p>Substituting their form correctly into recurrence relation.</p> <p>Deriving two equations in <math>a</math> and <math>b</math> using a correct method (eg comparing coefficients)</p> <p>Full form of GS, including <math>u_n =</math>, must be seen.</p>	<p>Condone missing brackets around <math>-2</math> unless misused.</p> <p>Other forms eg <math>an^2 + bn + c</math> are allowable provided <math>a = 0</math> derived.</p> <p>cao</p>
4	(b)	<p>Either: <math>n = 0 \Rightarrow 1 + \alpha + \beta = 6</math>            or: <math>n = 1 \Rightarrow 1 - 2 + 5\alpha - 2\beta = 10</math>  <math>\alpha + \beta = 5</math> and <math>5\alpha - 2\beta = 11</math>  <math>\Rightarrow 2\alpha + 2\beta = 10 \Rightarrow 7\alpha = 21</math></p> <p><math>\alpha = 3</math> and <math>\beta = 2</math> so  <math>u_n = 1 - 2n + 3 \times 5^n + 2 \times (-2)^n</math></p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1FT</b></p> <p><b>[3]</b></p>	<p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p>	<p>Substituting <math>n = 0</math> or <math>n = 1</math> in their GS to derive an equation in <math>\alpha</math> &amp; <math>\beta</math>.</p> <p>Deriving 2 equations from substituting <math>n = 0</math> &amp; 1, at least one correct for their GS, and attempting to solve .</p> <p>FT from their GS. Allow non-embedded values if GS seen in (a). Do not ISW.</p>	<p>This mark can be awarded if one of their equations is wrong.</p> <p>Attempt to solve can be implied by correct answer or valid algebra but incorrect answer with no working</p> <p><b>M0</b></p>
4	(c)	<p>From recurrence relation:  <math>u_2 = 3u_1 + 10u_0 + 24 \times 0 - 10</math>  <math>= 3 \times 10 + 10 \times 6 - 10 = 80</math>            From particular solution:  <math>u_2 = 1 - 2 \times 2 + 3 \times 5^2 + 2 \times (-2)^2</math>  <math>= 1 - 4 + 75 + 8 = 80</math></p>	<p><b>B1</b></p> <p><b>[1]</b></p>	<p><b>2.5</b></p>	<p>Both expressions properly seen (ie it must be clear that candidates are correctly using two different methods to find <math>u_2</math>).</p>	

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4	(d)	$v_n = \frac{1-2n}{p^n} + 3\left(\frac{5}{p}\right)^n + 2\left(\frac{-2}{p}\right)^n$ <p>If <math> p  &lt; 5</math> then <math>v_n \rightarrow \infty</math> while if <math> p  &gt; 5</math> then <math>v_n \rightarrow 0</math> as <math>n \rightarrow \infty</math></p> <p><math>p = 5</math> <math>q = 3</math></p>	<p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>[4]</b></p>	<p><b>3.1a</b></p> <p><b>2.1</b></p> <p><b>2.2a</b></p> <p><b>2.2a</b></p>	<p>Writing <math>v_n</math> in a form which enables the limit to be deduced.</p> <p>Convincing argument. FT for GS of the form: <math>c - dn + \alpha s^n + \beta t^n</math> (where <math> s  &gt;  t </math>).</p> <p>FT. <math>p = s</math> (must be a number). FT. <math>q = \alpha</math> (must be a number).</p>	<p>At most one of <math>c</math> and <math>d</math> is 0. <math>s</math> and <math>t</math> are not equal and both not 0. Both <math>\alpha</math> and <math>\beta</math> are not 0. Either <math> s  &gt; 1</math> or <math> t  &gt; 1</math> (or both). <b>A0</b> If <math>s = -t</math>. <b>A0</b> If <math>s = -t</math>. If <b>M0</b> then <b>SC2</b> for <math>p = 5, q = 3</math>.</p>
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5		$\frac{\partial g}{\partial x} = 2x \text{ or } \frac{\partial g}{\partial y} = 2y \text{ or } \frac{\partial g}{\partial z} = 4z$ $\nabla g = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} \text{ is the normal to the tangent plane at each point.}$ $\nabla g \cdot \mathbf{n} = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 4z$ $= \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cos \frac{\pi}{3}$ $= \sqrt{(2x)^2 + (2y)^2 + (4z)^2} \times 1 \times \frac{1}{2}$ $= 2\sqrt{x^2 + y^2 + 4z^2} \times \frac{1}{2} = \sqrt{126 - 2z^2 + 4z^2}$ $= \sqrt{126 + 2z^2}$ $\sqrt{126 + 2z^2} = 4z \Rightarrow 126 + 2z^2 = 16z^2$ $\Rightarrow 14z^2 = 126 \Rightarrow z^2 = 9 \Rightarrow z = \pm 3$ $z \geq 0 \Rightarrow z = 3 \text{ which is the equation of } \Pi.$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>[6]</b></p>	<p><b>3.1a</b></p> <p><b>3.1a</b></p> <p><b>2.2a</b></p> <p><b>1.1</b></p> <p><b>3.2a</b></p>	<p><math>g(x, y, z) = x^2 + y^2 + 2z^2</math> and surface is <math>g = 126</math>. Finding one correct partial derivative.</p> <p>Finding the normal vector.</p> <p>Dotting normal with normal to <math>x</math>-<math>y</math> plane.</p> <p>Expressing dot product in other form using correct value of angle.</p> <p>Using <math>\cos \frac{\pi}{3} = \frac{1}{2}</math>, forming magnitude of both normals and reducing to form <math>\sqrt{a+bz^2}</math> oe (could be done after squaring).</p> <p>Not <math>\pm</math> in final answer.</p>	<p>May be rewritten as</p> $z = f(x, y) = \sqrt{63 - \frac{1}{2}y^2 - \frac{1}{2}x^2}$ <p>but condone <math>\pm</math>.</p> $\nabla g = \begin{pmatrix} -\frac{x}{2z} \\ -\frac{y}{2z} \\ -1 \end{pmatrix} \text{ oe}$ <p>or <math>\sqrt{a+bx^2+by^2}</math> (could see eg <math>x^2 + y^2 = 108</math> oe after equating to <math>4z</math> and eliminating <math>z</math>).</p>
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6	(a)	$\frac{1}{(q+1)} + \frac{1}{(q+1)(q+2)} + \frac{1}{(q+1)(q+2)(q+3)} + \dots$ $< \frac{1}{(q+1)} + \frac{1}{(q+1)^2} + \frac{1}{(q+1)^3} + \dots$ $= \frac{1}{q+1} \left( 1 + \frac{1}{q+1} + \frac{1}{(q+1)^2} + \dots \right)$ $= \frac{1}{q+1} \cdot \frac{1}{1 - \frac{1}{q+1}}$ $= \frac{1}{q+1-1} = \frac{1}{q}$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p>[3]</p>	<p><b>2.1</b></p> <p><b>2.1</b></p> <p><b>2.1</b></p>	<p>Correct statement that given series is less than an infinite GP (could be eg <math>\frac{1}{q} + \frac{1}{q^2} + \dots</math> or <math>\frac{1}{3} + \frac{1}{3^2} + \dots</math>).</p> <p>FT on their <math>\frac{a}{1-r}</math>.</p> <p>AG. Intermediate step must be seen.</p>	
6	(b)	$q \geq 1 \Rightarrow \frac{1}{q} \leq 1$ <p>But <math>S &lt; \frac{1}{q} \Rightarrow S &lt; 1</math>; clearly <math>S &gt; 0</math> so <math>0 &lt; S &lt; 1</math> so <math>S \notin \square</math>.</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p>[2]</p>	<p><b>2.2a</b></p> <p><b>2.2a</b></p>	<p>AG. <math>S &gt; 0</math> must be stated but need not be justified.</p>	<p>Since <math>0 &lt; \frac{1}{q} \leq 1</math> and <math>S &lt; \frac{1}{q}</math> then <math>0 &lt; S &lt; 1</math> and <math>\therefore S \notin \square</math>.</p>
6	(c)	$e = \sum_{r=0}^{\infty} \frac{1}{r!} = \frac{p}{q} \Rightarrow eq! = \sum_{r=0}^{\infty} \frac{q!}{r!} = p(q-1)!$ $\therefore p(q-1)! = \sum_{r=0}^{\infty} \frac{q!}{r!} = \sum_{r=0}^q \frac{q!}{r!} + \sum_{r=q+1}^{\infty} \frac{q!}{r!}$ $= q! + q! + \frac{q!}{2!} + \dots + \frac{q!}{q!} + S$ $= 2q! + q(q-1)\dots \times 3 + q(q-1)\dots \times 4 + \dots + 1 + S$ <p><math>p(q-1)!</math> and <math>q! + q! + \frac{q!}{2!} + \dots + 1</math> are all integers but <math>S</math> is not which is a contradiction.</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>3.1a</b></p> <p><b>2.1</b></p> <p><b>3.2a</b></p>	<p>Multiplying both sides by <math>q!</math>. No need to mention <math>q \geq 1</math> in this part.</p> <p>Rewriting to a form in which it is clear that every term on both sides, except <math>S</math>, is an integer.</p> <p>AG</p>	

Y435/01

Mark Scheme

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		<p><b>Alternative Method:</b></p> $S = \sum_{r=q+1}^{\infty} \frac{q!}{r!}$ $S = q! \sum_{r=0}^{\infty} \frac{1}{r!} - \sum_{r=0}^q \frac{q!}{r!} = q!e - q! - \sum_{r=1}^q \frac{q!}{r!}$ $= p(q-1)! - q! - \sum_{r=1}^q q(q-1)\dots(q-r+1)$ <p>since <math>1 \leq r \leq q</math>  <math>p(q-1)!</math>, <math>q!</math> and <math>q(q-1)\dots(q-r+1)</math> are all integers but <math>S</math> is not which is a contradiction.</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Expressing <math>S</math> as an infinite sum in terms of factorials.</p> <p>Rewriting to a form in which it is clear that every term on both sides, except <math>S</math>, is an integer.</p> <p>AG</p>	
			[3]		

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